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Deciphering the properties of the medium produced in heavy ion collisions at RHIC by a pQCD analysis of quenched large $p_\perp$ π^0 spectra

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Abstract: We discuss the question of the relevance of perturbative QCD calculations for analyzing the properties of the dense medium produced in heavy ion collisions. Up to now leading order perturbative estimates have been worked out and confronted with data for quenched large $p_\perp$ hadron spectra. Some of them are giving paradoxical results, contradicting the perturbative framework and leading to speculations such as the formation of a strongly interacting quark-gluon plasma. Trying to bypass some drawbacks of these leading order analysis and without performing detailed numerical investigations, we collect evidence in favour of a consistent description of quenching and of the characteristics of the produced medium within the pQCD framework.

Keywords: Jets, QCD.

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1. Introduction

There is convincing evidence that hadron spectra at high $p_\perp$ in nuclear collisions are strongly suppressed [1, 2]. For neutral pion production in Au-Au collisions at RHIC the nuclear modification factor $R_{AA}(p_\perp)$ is measured to be about $0.2-0.3$ in the range of transverse momenta as large as $10\,GeV/c < p_\perp < 20\,GeV/c$ [3–5]. This quenching is usually attributed to radiative parton energy loss, for which the relevant expressions are obtained from perturbative QCD calculations [6, 7]. They are used to extract and analyze the properties of the deconfined medium produced in the collisions. As a consistency requirement these properties should be compatible with the perturbative framework.

However, the detailed analysis given in [8, 9] results in a (time-averaged) value for the transport coefficient $\hat{q}$ which characterizes the medium, exceeding $5\ GeV^2/fm$, almost a factor of 10 (or even more) bigger than a typical perturbative estimate at the energy density expected for $\sqrt{s_{NN}} = 200\,GeV$ Au-Au collisions!

An independent work [10], based on [11], calculating the quenching factor for hard pion production and comparing it with data measured at RHIC obeys this requirement of perturbative consistency: an averaged value of $\hat{q} \simeq 0.3 - 0.4\ GeV^2/fm$ is found, corresponding to an energy density $\epsilon \simeq 2\ GeV/fm^3$, as expected from pQCD for a deconfined equilibrated plasma. However, this value is based on imposing arbitrarily a mean path length for the jet of about $L = 5\,fm$, whereas for the denser medium discussed in [8] a characteristic path length of $L \simeq 2\,fm$ is obtained.

A related work on the nuclear modification factor for leading large $p_\perp$ hadrons (pions) [12], assuming a thermalized medium (and also taking into account absorption of thermal partons), shows that a value of R_{AA} is obtained which is compatible with measurements and the perturbative framework. This work based on the AMY [13] formalism describes the coherent gluon radiation, incorporating the Landau-Pomeranchuk-Migdal (LPM) effect in the range of gluon energies $\omega > \omega_{BH}$, where ω_{BH} (sometimes denoted by E_{LPM}) corresponds to the transition energy to the incoherent Bethe-Heitler radiation regime.

The analysis in [8], following the explicit calculations of "quenching weights" [14], does not distinguish between these two regimes. On the other hand, it does impose a kinematical constraint, taking properly into account the effect due to the transverse momentum phase space of the emitted gluon. This effectively constrains the soft LPM emission by imposing a lower energy cut-off $\hat{\omega}$, which depletes the gluon energy distribution.

In this note we argue that the introduction of this cut-off $\hat{\omega}$ is a priori not accurate enough because, as we shall see below, $\hat{\omega}$ is (much) smaller than ω_{BH} and thus not relevant for the LPM regime. This actually is one crucial reason for the large value of $\hat{q}$ found in [8]. We instead show that using ω_{BH} as the proper cut-off for the validity of soft LPM gluon emission may lead to values of $\hat{q}$ compatible with perturbative estimates.

The line of our arguments treating radiative energy loss follows the BDMPS [15, 16] - Zakharov [17, 18] - Wiedemann [19] approach, as it is reviewed in [6, 7]. The trigger bias induced by the steeply falling large $p_\perp$ vacuum production cross section of produced hadrons/neutral pions is treated as described in [20].

The infrared sensitivity of the quenching factor has already been commented upon in [20], where it is emphasized that the energy ω_{BH} plays a central role.

2. Limits on the quenching factor

As it is discussed in rather great detail in [20] the quenching effect is expressed by the factor

$$Q(p_\perp) \;=\; \int d\epsilon\, D(\epsilon)\; \left(\frac{d\sigma^{\rm vacuum}(p_\perp+\epsilon)/dp_\perp^2}{d\sigma^{\rm vacuum}(p_\perp)/dp_\perp^2}\right)\,, \tag{2.1}$$

where it is justified to express the probability $D(\epsilon)$ for emitting the energy ϵ into the medium by a Poissonian energy distribution

$$D(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!}\left[\prod_{i=1}^{n}\int d\omega_i\,\frac{dI(\omega_i)}{d\omega}\right]\delta\left(\epsilon-\sum_{i=1}^{n}\omega_i\right)\cdot \exp\left[-\int d\omega \frac{dI}{d\omega}\right]\,. \tag{2.2}$$

This expression is assumed to be valid for the emission of soft <u>*primary*</u> gluons. In the LPM regime $\omega \geq \omega_{BH}$ the bremsstrahlung spectrum $dI/d\omega$ is given in [16]. Correspondingly the multiplicity of LPM gluons with energies larger than ω is given by

$$N\left(\omega\right) \equiv \int_{\omega}^{\infty} d\omega'\,\frac{dI(\omega')}{d\omega'}\;. \tag{2.3}$$

For ω significantly less than the characteristic energy [14],

$$\omega_c = \frac{1}{2}\hat{q}L^2 \tag{2.4}$$

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but larger than ω_{BH}, the number of gluons is well approximated by

$$N(\omega) \simeq \frac{2\alpha_s C_R}{\pi} \left[\sqrt{\frac{2\omega_c}{\omega}} + \ln 2 \ln \frac{\omega}{\omega_c} - 1.44 \right] . \tag{2.5}$$

The following remarks are crucial for the subsequent analysis:

- The probability $D(\epsilon)$ is normalized by $\int d\epsilon D(\epsilon) = 1$.
- The ratio of cross sections in (2.1) is well approximated by

$$\frac{d\sigma^{\text{vacuum}}(p_\perp + \epsilon)/dp_\perp^2}{d\sigma^{\text{vacuum}}(p_\perp)/dp_\perp^2} \simeq \left(\frac{p_\perp}{p_\perp + \epsilon}\right)^n \simeq \exp\left(-\frac{n\epsilon}{p_\perp}\right) , \tag{2.6}$$

  when expressed in terms of an effective exponent n, which may depend on $p_\perp$, $n = n(p_\perp)$. In the following the approximation (2.6) is used.
- Concerning the underlying parton interactions one has to distinguish quark versus gluon jets. Since our concern is mainly neutral pion production at RHIC in the range $10 < p_\perp < 20\, GeV/c$, we effectively assume a dominating quark jet, radiating off (soft) gluons, therefore we take $C_R = C_F = 4/3$ in (2.5). This assumption is supported by the analysis in [8].
- The quenching factor $Q(p_\perp)$ corresponds to the experimentally measured ratio [5]

$$R_{AA}(p_\perp) = \frac{dN_{AA}}{< N_{coll} > dN_{NN}} , \tag{2.7}$$

  for central A-A collisions versus nucleon-nucleon (NN) collisions. In the following only neutral pion production at pseudo-rapidity $\eta = 0$ is considered.
- The transverse momenta of the produced pions are not asymptotically large, although only leading order pQCD calculations are considered.

Neglecting in the following the contribution from Bethe-Heitler emission (appendix A), the quenching factor due to LPM emission becomes

$$Q(p_\perp) \simeq \int_0^\infty d\epsilon\, D(\epsilon) \exp\left\{-\frac{n\epsilon}{p_\perp}\right\} = \exp\left\{-\int_{\omega_{BH}}^\infty \frac{dI}{d\omega}\left[1 - \exp\left(-\frac{n\omega}{p_\perp}\right)\right] d\omega\right\} , \tag{2.8}$$

where we take as a lower cut-off the energy ω_{BH}, which indicates the transition between the Bethe-Heitler and the LPM regime. By partial integration and with (2.3), the quenching factor becomes

$$Q(p_\perp) = \exp\left\{-N(\omega_{BH})\left[1 - \exp\left(-\frac{n\omega_{BH}}{p_\perp}\right)\right]\right\} \cdot \exp\left\{-\frac{n}{p_\perp}\int_{\omega_{BH}}^\infty N(\omega)\exp\left(-\frac{n\omega}{p_\perp}\right) d\omega\right\} . \tag{2.9}$$

As $N(\omega)$ decreases with increasing gluon energy [20], one finds the following two bounds for $Q(p_\perp)$:

$$Q_{min}(p_\perp) = \exp\left[-N(\omega_{BH})\right] , \tag{2.10}$$

i.e. by replacing $N(\omega) = N(\omega_{BH})$ in the integrand of the integral in (2.9), and

$$Q_{max}(p_\perp) \;=\; \exp\left\{-N(\omega_{BH})\left[1-\exp\left(-\frac{n\omega_{BH}}{p_\perp}\right)\right]\right\}, \tag{2.11}$$

i.e. by neglecting the second exponential factor of (2.9). So that, the experimental ratio is required to be bounded as follows,

$$Q_{min}(p_\perp) < R_{AA}(p_\perp) < Q_{max}(p_\perp)\;. \tag{2.12}$$

One notes that actually $Q_{min}(p_\perp)$ does not depend on $p_\perp$, and that $Q_{max}(p_\perp)$ approaches 1 for large $p_\perp \gg n\omega_{BH}$. For a fixed value of $p_\perp/n \simeq O(1\,GeV)$, the interval $Q_{max} - Q_{min}$ becomes rather tight, when ω_{BH} is not a small energy.

When comparing the quantities $Q_{min,max}$ with the ones defined in [8, 14] Q_{min} is related to the discrete weight of the probability distribution $D(\epsilon)$ by $p_0 = Q_{min}$, where, instead of the soft cut-off $\hat{\omega}$ the Bethe-Heitler one ω_{BH} has to be taken, as required by the LPM radiation formalism. According to our analysis Q_{max} is the relevant, important quantity, related to the continuous part of $D(\epsilon)$, which translates to $Q_{max} - Q_{min}$.

One may notice that the suppression factor $Q(p_\perp)$ of (2.9) does satisfy (for the same value of ω_c)

$$Q(p_\perp, \omega_{BH}) \geq Q(p_\perp, \hat{\omega})\,, \tag{2.13}$$

where the value of the lower cut-off is indicated. But here is where Q_{max} comes into the game. Its actual value and the ones of the various energy scales cannot be left out of the discussion !

Typical estimates of the medium characteristics as constrained by the experimental results are discussed in the next section. In particular we find that $\omega_{BH} \sim 1.5{-}2.0\,GeV$. As a consequence, for values of $p_\perp/n \sim 1\,GeV$, the bounds Q_{min}, Q_{max} are indeed constraining. On the contrary, taking $\hat{\omega}$ as the lower energy cut-off, one finds that due to $\hat{\omega} \ll \omega_{BH}$, Q_{min} and Q_{max} differ significantly and the relation (2.12) is no longer constraining.

Replacing ω_{BH} by $\hat{\omega}$ in the estimate for Q_{max}, the constraints are not very tight indeed: as an example when choosing $\hat{\omega} \simeq 0.45\;GeV$, guided by relation (3.6) to be derived in the next section, we find $Q_{max} \simeq 0.55$, instead of $Q_{max} \simeq 0.30$. In both cases $Q_{min} = 0.2$.

3. Kinematical and consistency constraints

As mentioned earlier, the detailed discussion given in [8] in order to determine the medium induced gluon radiation intensity $dI/d\omega$ takes into account the kinematical constraint associated to the transverse momentum phase space of the emitted gluon. This constraint is not implemented in earlier works [15]. The effect of the kinematical limitation is obtained by estimating the ratio $k_\perp/\omega$ in the LPM regime: in this coherent regime the transverse momentum $k_\perp$ of the emitted gluon may be given by

$$k_\perp^2 \simeq \frac{t_{coh}}{\lambda_g}\,\mu^2\,, \tag{3.1}$$

where μ is the typical transverse momentum transfer in a single scattering (i.e. the Debye mass screening the gluon exchange) and λ_g the gluon mean free path, such that t_{coh}/λ_g is the number of coherent scattering centers in the medium which the gluon encounters before being emitted after the time $t_{coh} \simeq 2\omega/k_\perp^2$. One finds

$$k_\perp^2 \simeq \sqrt{2\hat{q}\omega}\,, \tag{3.2}$$

where $\hat{q} \simeq \mu^2/\lambda_g$. As a consequence, since $k_\perp \leq \omega$, gluons have to be emitted dominantly above the energy $\hat{\omega}$ defined by

$$\hat{\omega} = (2\hat{q})^{1/3} = \omega_c \left(\frac{2}{R}\right)^{2/3}\,, \tag{3.3}$$

where it is convenient to introduce the dimensionless parameter [21, 22, 14]

$$R = \omega_c L = \frac{1}{2}\hat{q} L^3\ . \tag{3.4}$$

Now, we should take into account that the multiple scattering formalism used throughout the derivation of $dI/d\omega$ requires the condition [23, 15]

$$\lambda_g \mu \gg 1\ . \tag{3.5}$$

Using the fact that ω_{BH} may be expressed as $\omega_{BH} \simeq \lambda_g \mu^2$ we obtain the following parametric inequality

$$\frac{\hat{\omega}}{\omega_{BH}} \sim \frac{2^{1/3}}{(\lambda_g \mu)^{4/3}} \ll 1\ . \tag{3.6}$$

As a consequence of this inequality, the Bethe-Heitler energy remains the proper and relevant lower limit for the validity of the LPM gluon emission spectrum.

Let us summarize what is already known about the implementation of the various above mentioned constraints. In the BDMPS framework [15, 16] where no kinematical cut-off is imposed on the $k_\perp$ integration, i.e. in the limit $R \to \infty$, the gluon number only depends on the ratio ω/ω_c: $N(\omega) = N(\omega/\omega_c)$. As a consequence the resulting quenching factor $Q(p_\perp)$ is effectively a scaling function in the variable $X = p_\perp/(n\omega_c)$ [20], such that in the relevant analysis given in [10] only the characteristic gluon energy ω_c is extracted from the the neutral pion single-inclusive data measured by the PHENIX Collaboration in Au-Au collisions [3, 4], and found to be $\omega_c = 20 - 25\,GeV$. Moreover, as already mentioned, the path length is arbitrarily chosen so that the medium characteristics are not quantitatively constrained. In [8] the typical value of the parameter R relevant for the description of RHIC data on pion production at $\sqrt{s_{NN}} = 200\,GeV$ is estimated to be $R \simeq 1000$, equivalent to $\hat{\omega} \simeq 0.016\ \omega_c$.

4. Comparison with experiment

We shall first discuss a few semi-quantitative estimates of the parameters describing the medium extracted from the comparison with data within the framework described in the previous sections.

4.1 Averages

We concentrate on the data for $R_{AA}(p_\perp)$ with $p_\perp \geq 10\,GeV/c$: $R_{AA}(p_\perp) \simeq 0.2$, remaining essentially flat up to $p_\perp \simeq 20\,GeV/c$.

In order to start the discussion, we use these data to obtain values for Q_{min} and Q_{max}, corresponding to the experimental error bars. Taking $Q_{min} \simeq 0.2$ leads to $N(\omega_{BH}/\omega_c) \simeq 1.6$, which allows to estimate $\omega_{BH}/\omega_c \simeq 3.5 \cdot 10^{-2}$ when using (2.5) and $\alpha_s = 1/2$ (see also figure 1). When we take $p_\perp/n \simeq O(1\,GeV)$, observing that the experimental error bars allow us to fix the value of $Q_{max} < 0.3$, we deduce the typical value of ω_{BH}. From (2.11) we obtain $\omega_{BH} \geq 1.4\,GeV$, in agreement with thermal estimates (appendix B). Taking $\omega_{BH} \simeq 1.6\,GeV$, we find $\omega_c \simeq 45\ GeV$. We note that this value is bigger by a factor 2 than the one extracted in [10].

Depending on the typical average path length we derive estimates for the time $-$averaged transport coefficient,

$$\hat{q} = \frac{2\omega_c}{L^2} \simeq \frac{18}{(L[fm])^2}\,\frac{GeV^2}{fm}\,, \tag{4.1}$$

i.e. $\hat{q} \simeq 1.1\ GeV^2/fm$ for $L = 4\,fm$, and $\hat{q} \simeq 2.0\ GeV^2/fm$ for $L = 3\,fm$. This corresponds to values of $R \sim 900\ -\ 700$, not far from $R \simeq 1000$ in [8].

We observe that these estimates are indeed valid beyond the $R = \infty$ limit, since in the region of $\omega_{BH}/\omega_c > 3.5\ 10^{-2}$ the sensitivity on R, for $R \geq 1000$, becomes weak.

The cut-off ω_{BH} is effectively of order $1-2\ GeV$ whereas $\hat{\omega}$, as imposed by Eskola et al. [8] is also 1 GeV, when using relations given in [14]. One may wonder then why choosing one or the other is a crucial feature, independently of judging the validity of using the LPM spectrum away from $\omega > \omega_{BH}$. One way to understand qualitatively this fact is that in the analysis of ref. [14], the $\hat{\omega}$ cut-off is implemented effectively in the $k_\perp$ integration for the emitted gluon whereas here the ω_{BH} cut-off appears as an IR cut-off on the emitted gluon energy in the expression of $Q(p_\perp)$. That has consequences on the effectiveness of the cut-off ending up with the following estimates: on the one hand $\omega_{BH}/\omega_c \sim 3.5 \cdot 10^{-2}$, on the other hand for values of $R \sim 1000$, $\widehat{\omega}/\omega_c \sim 10^{-2}$ as inferred from eq. (3.3), thus leading to different values of ω_c and $\widehat{q}$ when taking a cut-off order of magnitude of 1 GeV.

In fact, what should be done is to implement both cut-offs in the calculation of the quenching weights of Ref [14]. The 1st one, ω_{BH}, is essential for the validity of the soft LPM gluon emission regime, the 2nd one, $\widehat{\omega}$, is much smaller than ω_{BH}, which indicates that the shape of the emission spectrum may not be too sensitive to this 2nd cut-off. This is what we have assumed in our semi-quantitative analysis but a more complete calculation should be performed.

Let us nonetheless comment further-at a semi-quantitative level-on the impact on the value of $\widehat{q}$ of the choice of $\widehat{\omega}$ rather than ω_{BH} as the cut-off.

Indeed the above indicates that although larger than expected from leading order estimates based on the presence of a thermalized and ideal QGP (appendix B), the values obtained above for $\hat{q}$ are much smaller than the ones quoted in [8], namely $5 < \hat{q} < 15\,GeV^2/fm$.

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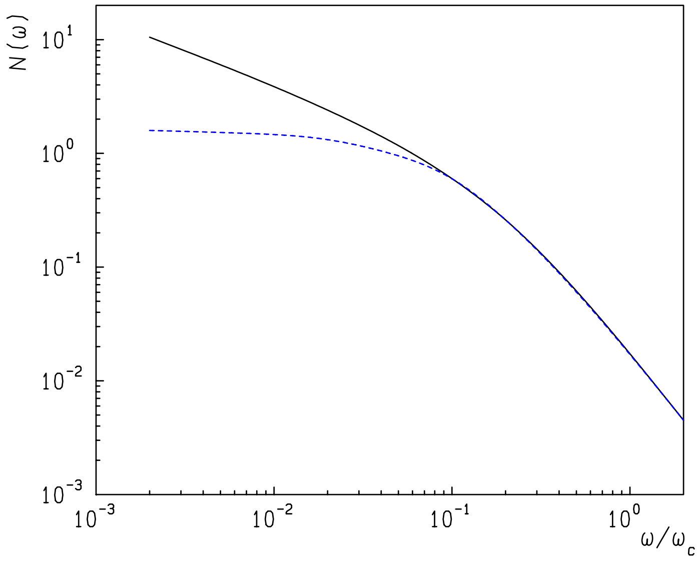

Figure 1: The gluon multiplicity as a function of ω, for $R = \infty$ (solid curve) and $R = 1000$ (dashed curve) taken from [14], but for $\alpha_s = 1/2$.

An even smaller value may be obtained with $L = 5\,fm$, namely $\hat{q} \simeq 0.7\,GeV^2/fm$. In a thermal gluonic system this implies an ($\underline{\text{time} - \text{averaged}}$) energy density of $\epsilon \simeq 4\,GeV/fm^3$.

It is obvious that the values of $\hat{q}$ and L are strongly correlated, namely a large transport coefficient, corresponding to a dense medium implies a shorter path length L, and vice versa. For a realistic average path length of $L \simeq 3\,fm$ in the case of Au-Au collisions under consideration the prefered value of the $\underline{\text{time} - \text{averaged}}$ transport coefficient becomes

$$\hat{q} \simeq 2\ GeV^2/fm\,, \tag{4.2}$$

which may still be accomodated into the pQCD framework, at least within the uncertainties of LO approximations, contrary to the "strong" QGP values of [8].

In fact, the actual values of the cut-off cannot be left out of the discussion. Taking numbers quoted above: $\hat{q} = 10\ GeV^2/fm$ and $L = 2\,fm$, leads to $\omega_c = 100\ GeV$. If as indicated above, we take $\hat{\omega}/\omega_c \simeq 10^{-2}$, one finds $\hat{\omega} \simeq 1\,GeV$, and thus correspondingly $\omega_{BH} \simeq 3-4\ GeV$, which is too large to make sense for energies/transverse momenta under consideration! If, on the other hand, we want to have a reasonable value of $\omega_{BH} \simeq 1.4\ GeV$, from the start, imposing correspondingly that the value of $\hat{\omega}$ is a factor $3-4$ smaller and keeping $\hat{\omega}/\omega_c \simeq 10^{-2}$, we find ω_c and thus $\hat{q}$, 3 -4 times smaller. We use the value of $Q_{max} \leq 0.3$ and $Q_{min} \simeq 0.2$ as a way to constrain the relevant parameters: from Q_{max}, we take the cut-off to be $\simeq 1.4\ GeV$. This cut-off can only be ω_{BH}. Then from $Q_{min} \simeq 0.2$, we deduce ω_c. Finally, we find, depending on the length L, reasonably small values of $\hat{q}$.

A determination of the average L should be possible with the expression for $Q_{min}(p_\perp)$, (2.10), which depends on L. Since $\omega_{BH} \ll \omega_c$, we use to a good approximation (2.5) to determine $N(\omega_{BH}) = N(\omega_{BH}/\omega_c)$ and insert (neglecting logarithmic factors)

$$\frac{2\omega_c}{\omega_{BH}} \simeq \frac{L^2}{\lambda_g^2} = \left(\frac{N_c}{C_F}\right)^2 \left(\frac{L}{\lambda_q}\right)^2 , \tag{4.3}$$

defining the quark mean free path $\lambda_q = \frac{N_c}{C_F}\lambda_g$, such that

$$Q_{\min}(p_\perp) \simeq \exp\left\{-\frac{2\alpha_s C_F}{\pi}\left[\frac{N_c}{C_F}\frac{L}{\lambda_q} + \ln 2 \ln\left(\frac{2\lambda_q^2 C_F^2}{L^2 N_c^2}\right) - 1.44\right]\right\} . \tag{4.4}$$

In leading order $L/\lambda_q \gg 1$ the dependence with respect to the path length has the typical characteristic behaviour of a survival probability $\exp[-L/\lambda_{eff}]$, where $\lambda_{eff} \simeq \frac{\pi}{2\alpha_s N_c}\lambda_q \simeq \lambda_q$ for $\alpha_s \simeq 1/2$. Without further geometrical restrictions the mean path length $\langle L\rangle$ would be just given by the mean free path of a quark jet in the medium, $\langle L\rangle \simeq \lambda_q$.

A better estimate of L/λ_q, however, is obtained by taking the full expression (4.4) into account: for $Q_{min} \simeq 0.2$ the corresponding ratio is $L/\lambda_q \simeq 3.35$.

In order to obtain L, we estimate the mean free path $\lambda_q = 9/4\lambda_g$ from $\hat{q}$ and ω_{BH} by

$$\lambda_q \simeq \frac{9}{4}\sqrt{\frac{\omega_{BH}}{\hat{q}}} \simeq 0.9 \;\; fm\,, \tag{4.5}$$

with $\hat{q} \simeq 2\,GeV^2/fm$ and $\omega_{BH} \simeq 1.6\,GeV$ and find $L \simeq 3\,fm$, consistent with the prefered value given above. Note the related estimate for the mass μ: $\mu \simeq 0.9\ GeV$.

4.2 Nuclear geometry

So far we have considered averaged values for Q_{min}, Q_{max} without taking the nuclear geometry explicitely into account. We now present a more detailed discussion which, as we shall show, leads on a firmer basis to similar conclusions as above as far as the medium parameters are concerned. We assume head-on collisions and essentially cylinder-like Au nuclei. The quark jet is produced in Au-Au collisions at mid-rapidity and propagates in the transverse plane. Following [8], one starts from the geometrical transverse path length

$$L_{\rm geom}(\vec{s}) = -s\cos\phi_{LS} + \sqrt{s^2\cos^2\phi_{LS} + R_{Au}^2 - s^2}, \tag{4.6}$$

where the position at which the parton is produced is denoted by the vector $\vec{s}$ in the transverse plane. ϕ_{LS} is the angle of propagation with respect to this vector. This geometrical picture allows us to obtain an average value for $\langle Q_{min}\rangle$ by calculating

$$\langle Q_{min}(\hat{q}/\omega_{BH})\rangle = \frac{\int d^2s \exp\{-N(L_{\rm geom})\}}{\pi R_{Au}^2}\,, \tag{4.7}$$

with $|\vec{s}| \leq R_{Au}$, the radius of the Au nucleus. In order to obtain $N(L_{\rm geom})$ we use (2.5) with $\omega_c/\omega_{BH} = \hat{q}/(2\omega_{BH})\ L_{\rm geom}^2$.

In figure 2 we plot $\langle Q_{min}\rangle$ as a function of $\hat{q}/\omega_{BH}$, and observe that $\langle Q_{min}\rangle \simeq 0.2$ for

$$\hat{q}/\omega_{BH} \simeq 1.4\ \frac{GeV}{fm}\ . \tag{4.8}$$

In the same figure 2 we plot $\langle Q_{max}\rangle$, obtained analogously to (4.7), for different values of ω_{BH} (and for $n/p_\perp = 1/GeV$).

In order to have $\langle Q_{max}\rangle < 0.3$ - together with $\langle Q_{min}\rangle \simeq 0.2$ - we find $0.75 < \omega_{BH} < 2.0\ GeV$.

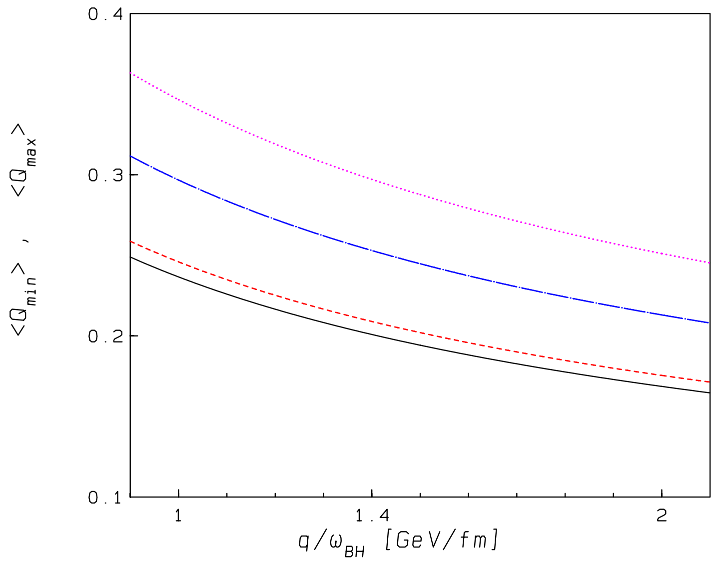

Figure 2: $\langle Q_{min}\rangle$ (solid curve) and $\langle Q_{max}\rangle$, respectively, as functions of $\hat{q}/\omega_{BH}$ according to (4.7). $\langle Q_{max}\rangle$ for different values of ω_{BH}:0.75 (dotted), 1.15 (dashed-dotted) and 2.75 GeV (dashed curve).

In figure 3 we show $\langle Q_{min}\rangle$, but this time as a function of λ_q, obtained from

$$\langle Q_{min}(\lambda_q)\rangle = \frac{\int d^2s\ Q_{min}(L_{\rm geom}/\lambda_q)}{\pi R_{Au}^2}\,, \tag{4.9}$$

after inserting $L = L_{geom}$ in (4.4). This way we find $\lambda_q \simeq 0.85\ \ fm\ \ (\lambda_g \simeq 0.38\ \ fm)$, when $Q_{min} \simeq 0.2$, in good agreement with (4.5).

From figure 3 we deduce that $0.65 < \mu < 1.1\ GeV$. Finally, within these bounds the transport coefficient $\hat{q} \simeq \mu^2/\lambda_g$ becomes

$$\hat{q} \simeq 1.0 - 3.0\ GeV^2/fm\ . \tag{4.10}$$

Averaging with respect to the nuclear geometry in this straightforward manner leads to a final estimate which is compatible with (4.2). The "uncertainty" given by (4.10) may be considered as "theoretical error" on the derived medium parameters.

Concerning the robustness of the estimate (4.10) for $\hat{q}$, one has to keep in mind that besides the IR sensitivity under discussion, all the estimates are based on LO QCD calculations. Nevertheless, the explanation of quenching as being due to the LPM radiation in a perturbative regime appears to be a robust statement.

A value of $\hat{q} \simeq 1.8\ GeV^2/fm$ would correspond to a temperature of $T \simeq 375\,MeV$, corresponding to an average energy density of $\epsilon \simeq 12.5\ GeV/fm^3$, in agreement with the results quoted in [12].

In order to obtain the actual value of $\hat{q}$ at the very early stage of the collisions, at times $\tau \simeq 1/p_\perp < 2\cdot 10^{-2}\,fm$, one has to include the effects due to the longitudinal [24, 22], but also transverse [25] expansion of the dense system during the time of $O(L)$, the jet takes to propagate through this medium.

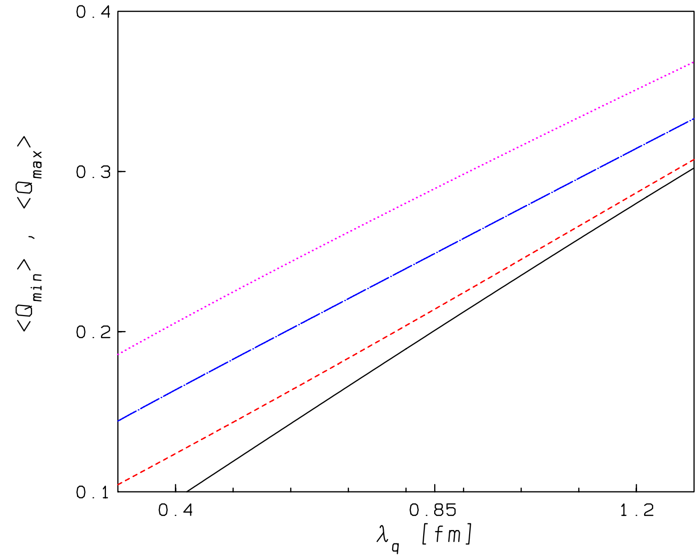

Figure 3: $\langle Q_{min}\rangle$ (solid curve) and $\langle Q_{max}\rangle$, respectively, as functions of λ_q according to (4.9). $\langle Q_{max}\rangle$ for different values of the screening mass μ: 0.65 (dotted), 0.8 (dashed-dotted) and 1.1 GeV (dashed curve).

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A. Estimate of the Bethe-Heitler and absorption contributions

In the plausible context, where the medium is thermalized, let us investigate how the contributions of the Bethe-Heitler and the absorption spectra modify the analysis discussed so far in the paper.

In LO pQCD the Bethe-Heitler-Gunion-Bertsch [26] differential spectrum for inclusive gluon production in a medium of length L in the presence of L/λ_g scatterers reads

$$\frac{dI}{dyd^2p_\perp} = \frac{\alpha_s C_F}{\pi^2}\frac{1}{p_\perp^2}\left(\frac{L}{\lambda_g}\right), \tag{A.1}$$

which after $p_\perp$-integration becomes

$$\omega\frac{dI}{d\omega}\bigg|_{BH} \simeq \frac{\alpha_s C_F}{\pi}\ln\frac{\omega_{BH}^2}{\omega_{\rm cut}^2}\left(\frac{L}{\lambda_g}\right) . \tag{A.2}$$

Because of the presence of the (non-perturbative) IR-cut, $\omega_{\rm cut}$, this BH-intensity is not precisely determined. Nevertheless, an estimate of the gluon energy ω_{BH} may be obtained from matching (A.2) with the LPM expression for the intensity at $\omega = \omega_{BH}$,

$$\omega\frac{dI}{d\omega}\bigg|_{LPM} \simeq \frac{\alpha_s C_F}{\pi}\sqrt{\frac{2\omega_c}{\omega_{BH}}}, \tag{A.3}$$

valid for $\omega \ll \omega_c$. When logarithmic factors are taken to be of $O(1)$, i.e. $\omega_{BH} \simeq 1.65\, \omega_{\rm cut}$, we find

$$\omega_{BH} \simeq \mu^2 \lambda_g, \tag{A.4}$$

in terms of the screening mass μ and the mean free path of the gluon λ_g.

The resulting quenching factor becomes

$$Q_{BH}(p_\perp) = \exp\left\{ - \int_{\omega_{\rm cut}}^{\omega_{BH}} \left.\frac{dI}{d\omega}\right|_{BH} \left[1 - e^{n\omega/p_\perp}\right] \right\}, \tag{A.5}$$

and expanding the integrand in the limit of small ω, we find with (A.2)

$$Q_{BH}(p_\perp) \geq \exp\left\{ - \frac{\alpha_s C_F}{\pi} \left(\frac{L}{\lambda_g}\right) \frac{n}{p_\perp} (\omega_{BH} - \omega_{\rm cut}) \right\} . \tag{A.6}$$

Following [12], we consider the absorption contribution in the presence of a heat bath. It is assumed that for the radiation energy $\omega < 0$ the spectrum is approximated by

$$\left.\frac{dI}{d\omega}\right|_{abs} \simeq \frac{\alpha_s C_F}{\pi} \frac{1}{|\omega|} \left(\frac{L}{\lambda_g}\right) e^{-|\omega|/T}, \tag{A.7}$$

i.e. the Bethe-Heitler spectrum multiplied by a Boltzmann factor with temperature T. The corresponding quenching factor $Q_{abs}(p_\perp)$ then becomes

$$\begin{aligned} Q_{abs}(p_\perp) &= \exp\left\{ - \int_0^\infty \frac{dI}{d|\omega|} \left[1 - e^{n|\omega|/p_\perp}\right] \right\} \\ &= \exp\left\{ - \frac{\alpha_s C_F}{\pi} \frac{L}{\lambda_g} \int_0^\infty \frac{d\omega}{\omega} e^{-\omega/T} \left[1 - e^{n\omega/p_\perp}\right] \right\}. \end{aligned} \tag{A.8}$$

The integral may be approximated by

$$\int_0^\infty \frac{d\omega}{\omega} e^{-\omega/T} \left[1 - e^{n\omega/p_\perp}\right] \simeq -\frac{n}{p_\perp} \int_0^\infty d\omega e^{-\omega/T} = -\frac{nT}{p_\perp}, \tag{A.9}$$

leading to

$$Q_{abs}(p_\perp) \geq \exp\left\{ \frac{\alpha_s C_F}{\pi} \left(\frac{L}{\lambda_g}\right) \frac{nT}{p_\perp} \right\}. \tag{A.10}$$

Finally, multiplying the two quenching factors $Q_{BH}(p_\perp)$ and $Q_{abs}(p_\perp)$, leads to a lower bound

$$Q_{BH}(p_\perp) Q_{abs}(p_\perp) \geq \exp\left\{ + \frac{\alpha_s C_F}{\pi} \left(\frac{L}{\lambda_g}\right) \frac{n}{p_\perp} \left[T - (\omega_{BH} - \omega_{\rm cut})\right] \right\} . \tag{A.11}$$

Taking as typical numbers: $\omega_{BH} \simeq 1.6\, GeV$, $\omega_{\rm cut} \simeq 1\ GeV$, $T = 350\ MeV$, $p_\perp = 10\, GeV$, $n = 10$, and $L = 3\, fm$, $\lambda_g = 0.38\, fm$, we find

$$Q_{BH}(p_\perp) Q_{abs}(p_\perp) \geq 0.7\ . \tag{A.12}$$

This indicates that we may as a first guess, as already suggested in [12], neglect altogether the Bethe-Heitler and absorption processes, meaning that the values we thus obtain for $\hat{q}$ are in fact upper bounds, and therefore comforting the perturbative framework.

B. Thermal parameters

Let us consider an equilibrated system ($N_c = 3, N_f = 0$) in the weak coupling QCD limit [27] and give a short summary of the elements which enter our analysis.

In LO given the temperature T the screening mass is

$$\mu^2 = 4\pi\alpha_s T^2. \tag{B.1}$$

The gluon mean free path λ_g is expressed in terms of the gluon density

$$\rho_g = \frac{16}{\pi^2}\zeta(3)T^3\ , \quad \zeta(3) = 1.202\,, \tag{B.2}$$

and the (transport) gluon-gluon cross section (to logarithmic accuracy)

$$\sigma_T^{gg} \simeq \frac{N_c}{C_F}\frac{2\pi\alpha_s^2}{\mu^2}\ln\left(1/\alpha_s\right) \simeq \frac{9\pi\alpha_s^2}{2\mu^2}, \tag{B.3}$$

when neglecting logarithmic dependence, i.e.

$$1/\lambda_g = \rho_g \sigma_T^{gg} \simeq \frac{18}{\pi^2}\zeta(3)\alpha_s T \simeq 2.2\ \alpha_s T\ . \tag{B.4}$$

The mean free path for a quark is $\lambda_q = 9/4\lambda_g$. The corresponding energy density of this gluonic system is

$$\epsilon = \frac{8\pi^2}{15}T^4. \tag{B.5}$$

Typical orders of magnitude may be given e.g for a temperature of $T = 400\ MeV$ and a coupling $\alpha_s = 1/2$: the screening mass is $\mu \simeq 1\ GeV$, the mean free path $\lambda_g \simeq 0.45\ \ fm\ \ (\lambda_q \simeq 1\ \ fm)$, implying $\omega_{BH} \simeq \mu^2\lambda_g \simeq 2.25\ GeV$. The transport coefficient is estimated as $\hat{q} \simeq \mu^2/\lambda_g \simeq 2.2\ GeV^2/\,fm$, leading to an energy density of $\epsilon \simeq 17\ GeV/\,fm^3$ when using $\hat{q}\ \simeq 2\ \epsilon^{3/4}$ [28].

References

[1] BRAHMS collaboration, I. Arsene et al., *Quark gluon plasma and color glass condensate at RHIC? The perspective from the BRAHMS experiment*, *Nucl. Phys.* **A 757** (2005) 1 [nucl-ex/0410020];
PHENIX collaboration, K. Adcox et al., *Formation of dense partonic matter in relativistic nucleus nucleus collisions at RHIC: experimental evaluation by the PHENIX collaboration*, *Nucl. Phys.* **A 757** (2005) 184 [nucl-ex/0410003];
B.B. Back et al., *The PHOBOS perspective on discoveries at RHIC*, *Nucl. Phys.* **A 757** (2005) 28 [nucl-ex/0410022];
STAR collaboration, J. Adams et al., *Experimental and theoretical challenges in the search for the quark gluon plasma: the STAR collaboration's critical assessment of the evidence from RHIC collisions*, *Nucl. Phys.* **A 757** (2005) 102 [nucl-ex/0501009].

[2] B. Müller, *Quark matter 2005: theoretical summary*, nucl-th/0508062;
C.A. Salgado, *RHIC results from LHC perspectives*, *Nucl. Phys.* **A 774** (2006) 267 [hep-ph/0510062].

JHEP09(2006)059

[3] PHENIX collaboration, Y. Akiba, *Probing the properties of dense partonic matter at RHIC*, *Nucl. Phys.* **A 774** (2006) 403 [nucl-ex/0510008].

[4] T. Isobe, *Measurement of neutral pions in* $\sqrt{s_{nn}} = 200 - GeV$ *and* $62.4 - GeV\, Au + Au$ *collisions at RHIC-PHENIX*, nucl-ex/0510085.

[5] M.J. Tannenbaum, *Recent results in relativistic heavy ion collisions: from 'a new state of matter' to 'the perfect fluid'*, *Rept. Prog. Phys.* **69** (2006) 2005–2060 [nucl-ex/0603003].

[6] R. Baier, D. Schiff and B.G. Zakharov, *Energy loss in perturbative QCD*, *Ann. Rev. Nucl. Part. Sci.* **50** (2000) 37–69 [hep-ph/0002198].

[7] A. Kovner and U.A. Wiedemann, *Gluon radiation and parton energy loss*, hep-ph/0304151.

[8] K.J. Eskola, H. Honkanen, C.A. Salgado and U.A. Wiedemann, *The fragility of high-*p_T *hadron spectra as a hard probe*, *Nucl. Phys.* **A 747** (2005) 511 [hep-ph/0406319].

[9] C.A. Salgado, *Jet quenching*, hep-ph/0506008.

[10] F. Arleo, *Hard pion and prompt photon at RHIC, from single to double inclusive production*, hep-ph/0601075.

[11] F. Arleo, *Tomography of cold and hot QCD matter: tools and diagnosis*, *JHEP* **11** (2002) 044 [hep-ph/0210104].

[12] S. Turbide, C. Gale, S. Jeon and G.D. Moore, *Energy loss of leading hadrons and direct photon production in evolving quark-gluon plasma*, *Phys. Rev.* **D 72** (2005) 014906 [hep-ph/0502248].

[13] P. Arnold, G.D. Moore and L.G. Yaffe, *Photon and gluon emission in relativistic plasmas*, *JHEP* **06** (2002) 030 [hep-ph/0204343].

[14] C.A. Salgado and U.A. Wiedemann, *Calculating quenching weights*, *Phys. Rev.* **D 68** (2003) 014008 [hep-ph/0302184].

[15] R. Baier, Y.L. Dokshitzer, A.H. Mueller, S. Peigne and D. Schiff, *Radiative energy loss of high energy quarks and gluons in a finite-volume quark-gluon plasma*, *Nucl. Phys.* **B 483** (1997) 291 [hep-ph/9607355].

[16] R. Baier, Y.L. Dokshitzer, A.H. Mueller and D. Schiff, *Medium-induced radiative energy loss: equivalence between the BDMPS and Zakharov formalisms*, *Nucl. Phys.* **B 531** (1998) 403 [hep-ph/9804212].

[17] B.G. Zakharov, *Radiative energy loss of high energy quarks in finite-size nuclear matter and quark-gluon plasma*, *JETP Lett.* **65** (1997) 615 [hep-ph/9704255].

[18] B.G. Zakharov, *Radiative parton energy loss and jet quenching in high- energy heavy-ion collisions*, *JETP Lett.* **80** (2004) 617 [hep-ph/0410321].

[19] U.A. Wiedemann, *Jet quenching versus jet enhancement: a quantitative study of the BDMPS-Z gluon radiation spectrum*, *Nucl. Phys.* **A 690** (2001) 731 [hep-ph/0008241].

[20] R. Baier, Y.L. Dokshitzer, A.H. Mueller and D. Schiff, *Quenching of hadron spectra in media*, *JHEP* **09** (2001) 033 [hep-ph/0106347].

[21] R. Baier, Y.L. Dokshitzer, A.H. Mueller and D. Schiff, *Angular dependence of the radiative gluon spectrum and the energy loss of hard jets in QCD media*, *Phys. Rev.* **D 60** (1999) 064902 [hep-ph/9907267]; *On the angular dependence of the radiative gluon spectrum*, *Phys. Rev.* **D 64** (2001) 057902 [hep-ph/0105062].

[22] C.A. Salgado and U.A. Wiedemann, *A dynamical scaling law for jet tomography*, *Phys. Rev. Lett.* **89** (2002) 092303 [hep-ph/0204221].

[23] M. Gyulassy and X.-N. Wang, *Multiple collisions and induced gluon bremsstrahlung in QCD*, *Nucl. Phys.* **B 420** (1994) 583 [nucl-th/9306003].

[24] R. Baier, Y.L. Dokshitzer, A.H. Mueller and D. Schiff, *Radiative energy loss of high energy partons traversing an expanding QCD plasma*, *Phys. Rev.* **D 58** (1998) 1706 [hep-ph/9803473].

[25] T. Renk and J. Ruppert, *Flow dependence of high p_T parton energy loss in heavy-ion collisions*, *Phys. Rev.* **D 72** (2005) 044901 [hep-ph/0507075].

[26] J.F. Gunion and G. Bertsch, *Hadronization by color bremsstrahlung*, *Phys. Rev.* **D 25** (1982) 746.

[27] M. Le Bellac, *Thermal field theory*, Cambridge Univ. Press, Cambridge, 1996.

[28] R. Baier, *Jet quenching*, *Nucl. Phys.* **A 715** (2003) 209 [hep-ph/0209038].

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